(18)

[This question paper contains 4 printed pages]

**Your Roll No.** : 2019

**Sl. No. of Q. Paper** : **7463** **J**

Unique Paper Code : 32351301

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Theory of Real Functions

Semester : III

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **three** parts from each question.

(c) **All** questions carry equal marks.

1. (a) Find the following limit and establish it by using $\epsilon-\delta$ definition of limit :

$$\lim_{x \to -1} \frac{x+5}{2x+3}$$

(b) State and prove the sequential criterion for limits of a real valued function.

(c) Determine whether the following limit exists in R :

$$\lim_{x \to 0} \operatorname{sgn}\left(\sin 1/x^2\right)$$

(d) Show that :

$$\lim_{x \to 0^-} e^{\frac{1}{x}} = 0$$

and establish that

$$\lim_{x \to 0^+} e^{\frac{1}{x}}$$

does not exist in R .

2. (a) Let $c \in R$ and f be defined on $(c, \infty)$ and $f(x) > 0$ for all $x \in (c, \infty)$. Show that

$$\lim_{x \to \infty} f(x) = \infty$$

if and only if

$$\lim_{x \to \infty} \frac{1}{f(x)} = 0$$

(b) Evaluate the following limit by using the appropriate definition :

$$\lim_{x \to 1^+} \frac{x}{x-1}$$

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(c) Determine the points of continuity of the function f (x) = x [x] where [.] denotes the greatest integer function.

(d) A function $f : R \to R$ is such that $f(x+y) = f(x) + f(y)$ for all x,y in R. Prove that if f is continuous at some point $x_0$, then it is continuous at every point of R.

**3.** (a) Let $A \subseteq R$ and $f : A \to R$ and let $f(x) \geq 0$, for all $x \in A$. Let $\sqrt{f}$ be defined as $\sqrt{f}(x) = \sqrt{f(x)}$ for $x \in A$. Show that if f is continuous at a point $c \in A$, then $\sqrt{f}$ is continuous at c.

(b) Suppose that $f : R \to R$ is continuous on R and that f(r) = 0 for every rational number r. Prove that f(x) = 0 for all $x \in R$.

(c) Let f be a continuous and real valued function defined on a closed and bounded interval [a, b]. Prove that f is bounded. Give an example to show that the condition of boundedness of the interval cannot be dropped.

(d) State the intermediate value theorem. Show that $x_2^{k} = 1$ for some $x \in ]0, 1[$.

4. (a) Show that the function $f(x) = x^2$ is uniformly continuous on $[-2, 2]$, but it is not uniformly continuous on $R$.

(b) Prove that if f and g are uniformly continuous on $A \subseteq R$ and if they both are bounded on A, then their product fg is uniformly continuous on A.

(c) Show that the function

$f(x) = |x + 1| + |x - 1|$

is not differentiable at -1 and 1.

(d) Prove that if $f : R \to R$ is an even function and has a derivative at every point, then the derivative f' is an odd function.

5. (a) State Darboux theorem. Let I be an interval and $f : I \to R$ be differentiable on I. Show that if the derivative $f'$ is never zero on $I$, then either $f'(x) > 0$ for all $x \in I$ or $f'(x) < 0$ for all $x \in I$.

(b) Find the Taylor's series for cos x and indicate why it converges to cos x for all $x \in R$.

(c) Prove that $e^x \geq 1 + x$ for all $x \in R$, with equality occurring if and only if $x = 0.5$.

(d) Is $f(x) = |x|$, $x \in R$, a convex function ? Is every convex function differentiable ? Justify your answer.

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[This question paper contains 4 printed pages]

**Your Roll No.** : 2019

**Sl. No. of Q. Paper** : **7464** **J**

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Unique Paper Code : 32351302

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Group Theory - I

Semester : III

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) All questions carry equal marks.

1. (a) Let $G=\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in R, a \neq 0\right\}$. Show tha. G is a group under matrix multiplication.

(b) Let G be a group and H be a subset of G. Prove that H is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$. Hence prove that $H = \{A \in G : \det A$ is a power of $3\}$ is a subgroup of GL (2, R).

P.T.O.

(c) (i) Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have ?

(ii) If $|a| = n$ and k divides n, prove that $|a^{n/k}|=k$.

$6 \times 2 = 12$

2. (a) Let G = <a> be a cyclic group of order n. Prove that G = $\langle a^k \rangle$ if and only if gcd (k, n) =1. List all the generators of $Z_{20}$.

(b) (i) If a cyclic group has an element of infinite order, how many elements of finite order does it have.

(ii) List all the elements of order 6 and 8 in $Z_{30}$.

(c) Suppose that a and b are group elements that commute and have orders m and n. If $\langle a \rangle \cap \langle b \rangle = \{e\}$, Prove that the group contains an element whose order is the least common multiple of m and n. Show that this need not be true if a and b do not commute.

$6.5 \times 2 = 13$

3. (a) Let G be a group. Is $H = \{x^2 : x \in G\}$ a subgroup of G ? Justify.

(b) Prove that any two left cosets of a subgroup H in a group G are either equal or disjoint.

(c) Show that (Q ,+) has no proper subgroup of finite index.

$6 \times 2 = 12$

4. (a) Prove that every subgroup of index 2 is normal. Show that $A_5$ is normal subgroup of $S_5$.

(b) Let G be a group and H be a normal subgroup of G. Prove that the set of all left cosets of H in G forms a group under the operation aH.bH = abH where $a, b \in G$.

(c) If H is a normal subgroup of G with $|H| = 2$, prove that $H \subseteq Z(G)$. Hence or otherwise show that $A_5$ cannot have a normal subgroup of order 2.

6.5 × 2 = 13

5. (a) Let C be the set of complex numbers and

$$M = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in R \right\}.$$

Prove that C and M are isomorphic under addition and C* = C\{0} and M* = M\{0} are isomorphic under multiplication.

(b) Prove that a finite cyclic group of order n is isomorphic to the group $Z_n = \{0, 1, 2, \dots, n-1\}$ under addition modulo $n$.

(c) (i) Suppose that φ is an isomorphism from a group G onto a group G*. Prove that G is cyclic if and only if G* is cyclic.



P.T.O.

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(ii) Show that Z, the group of integers under addition is not isomorphic to Q, the group of rationals under addition. $6 \times 2 = 12$

6. (a) Let $\varphi$ be a group homomorphism from a group G to a group $G^*$ then prove that :

(i) $|\varphi(x)|$ divides $|x|$, for all x in G.

(ii) $\varphi$ is one-one if and only if $|\varphi(x)| = |x|$, for all x in G.

(b) State and prove the Third Isomorphism Theorem.

(c) (i) Let G be a group. Prove that the mapping $\phi(g) = g^{1}$, for all $g \in G$, is an isomorphism on G if and only if G is Abelian.

(ii) Determine all homomorphisms from $Z_n$ to itself.

$6.5 \times 2 = 13$

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[This question paper contains 7 printed pages]

**Your Roll No.** : 2019

**Sl. No. of Q. Paper** : **7465** **J**

Unique Paper Code : 32351303

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Multivariate Calculus

Semester : III

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** Sections are compulsory.

(iii) Attempt any **five** questions from each **Section**.

(iv) All questions carry equal marks.

P.T.O.

## Section- I

1. Given that the function

$$f(x,y)=\begin{cases}\dfrac{3x^3-3y^3}{x^2-y^2} & \text{for } x^2 \neq y^2\\ B & \text{otherwise}\end{cases}$$

is continuous at the origin, what is B ?

2. In physics, the *wave equation* is :

$$\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is :

$$\frac{\partial z}{\partial t}=c^2\frac{\partial^2 z}{\partial x^2}$$

Determine whether $z = \sin 5ct \cos 5x$ satisfies the wave equation, the heat equation, or neither.

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3. The radius and height of a right circular cone are measured with errors of at most 3% and 2%, respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.

4. If $f(x, y, z) = xy^2e^{xz}$ and $x = 2 + 3t$, $y = 6 - 4t$, $z = t^2$. Compute $\frac{df}{dt}(1)$.

5. Sketch the level curve corresponding to $C = 1$ for the function $f(x,y) = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ and find a unit normal vector at the point $P_0(2\sqrt{3})$.

6. Find the point on the plane $2x + y - z = 5$ that is closest to the origin.

## Section - II

7. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

8. Sketch the region of integration and then compute the integral $\int_0^1 \int_x^{2x} e^{y-x} \, dy \, dx$ in 2 ways :

   (a) with the given order of integration

   (b) with the order of integration reversed

9. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2} \, dy \, dx$ by converting to polar coordinates.

10. Find the volume of the tetrahedron bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.

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**11.** Compute $\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

**12.** Use the change of variables to compute $\iint_D \frac{(x-y)^4}{(x+y)^4} dydx$, where D is the triangular region bounded by the line $x + y = 1$ and the coordinate axes.

## Section - III

**13.** Find the work done by the force field $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\vec{i} - \frac{y}{\sqrt{x^2+y^2}}\vec{j}$ when an object moves from (a,0) to (0,a) on the path $x^2+y^2=a^2$.

**14.** Verify that the following line integral is independent of the path $\oint(3x^2+2x+y^2)$ dx + $(2xy+y^3)$ dy where C is any path from (0,0) to (0,1).



P.T.O.

**15.** Use Green's theorem to evaluate $\oint_C \left(x \sin x dx - \exp\left(y^2\right) dy\right)$ where C is the closed curve joining the points (1,-1) (2,5) and (-1,-1) in counterclockwise direction.

**16.** State Stoke's theorem and use it to evaluate $\iint_S \operatorname{curl} \vec{F}.d\vec{S}$ where $\vec{F} = xz\vec{i} + yz\vec{j} + xy\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

**17.** Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F}.\vec{N}dS$, where $\vec{F} = \left(x^2 + y^2 - z^2\right)\vec{i} + yx^2\vec{j} + 3z\vec{k}$ ; S is the surface comprised of the five faces of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ , missing $z = 0$.

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**18.** Evaluate $\iint_S 2x\,dS$ where S is the portion of the plane $x + y + z = 1$ with $x \geq 0,\ y \geq 0,\ z \geq 0$.

Sl-No of Q.P: 8898 2019

Unique Paper Code : 235301 (21)

Name of the Paper : MAHT 301-Calculus- II

Name of the Course : B.Sc. (Hons.) Mathematics- II

Semester : III

Duration : 3 hours

Maximum Marks : 75

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**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **five** questions from each Section.
4. **All** questions carry equal marks.

## SECTION - I

1. (a) Let $f$ be the function defined by :

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for, } (x,y) = (0,0) \end{cases}$$

Is $f$ continuous at (0,0)? Explain.

(b) The **Cauchy-Riemann equations** are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

where $u = u(x,y)$ and $v = v(x,y)$. Check if $u = e^{-x}cosy$, $v = e^{-x}siny$ satisfy the Cauchy-Riemann equations?

2. In physics, the *wave equation* is

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is

$$\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Determine whether $z = e^{-1}\left(sin\frac{x}{c} + cos\frac{x}{c}\right)$ satisfies the wave equation, the heat equation, or neither.

3. An open box has length 3ft, width 1ft, and height 2 ft and is constructed from material that costs \$2/ft$^2$ for the sides and \$3/ft$^2$ for the bottom. Compute the cost of constructing the box, and then use increments to estimate the change in cost if the length and width are each increased by 3 in. and the height is decreased by 4 in.

4. If $z = u + f(uv)$, show that

$$u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = u$$

5. (a) Find the directional derivative of $f(x, y) = ln(x^2 + y^3)$ at $P_0(1, -3)$ in the direction of $v = 2\mathbf{i} - 3\mathbf{j}$.

(b) Sketch the level curve corresponding to $C = 1$ for the function $f(x, y) = x^2 - y^2$ and find a normal vector at the point $P_0(2, \sqrt{3})$.

6. Find the maximum and minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ over the triangle with vertices (0,0), (9,0) and (0,9).

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## SECTION-II

7. Evaluate $\iint \frac{dA}{1+y^2}$ over a triangle D bounded by $x = 2y, y = -x$ and $y = 2$.

8. Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}}\, dxdy$ by converting to polar coordinates.

9. Find the volume V of the tetrahedron T bounded by the plane $x + y + z = 1$ and coordinate planes $x = 0, y = 0$ and $z = 0$.

10. Use spherical coordinates to evaluate $\iiint_D \frac{dxdydz}{\sqrt{x^2 + y^2 + z^2}}$ where D is the region given by $x^2 + y^2 + z^2 \leq 3, z \geq 0$.

11. Compute the integral $\iiint (x^4 + 2x^2y^2 + y^4)dxdydz$ over the cylindrical solid

$x^2 + y^2 \leq a^2$ with $0 \leq z \leq \frac{1}{\pi}$.

12. Use suitable change of variables to find the area of the region R bounded by the

hyperbolas $xy = 1$ and $xy = 4$ and lines $y = x$ and $y = 4x$.

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## SECTION-III

13. A force field in the plane is given by $\mathbf{F} = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$. Find the total work done by this force in moving a point mass counterclockwise around the square with vertices (0,0), (2,0), (2,2), (0,2).

14. (a) Show that the force field $\mathbf{F} = \sin z\,\mathbf{i} - z \sin y\mathbf{j} + (x\cos z + \cos y)\mathbf{k}$ is conservative.

(b) Verify that $\int_C [(3x^2 + 2x + y^2)dx + (2xy + y^3)dy]$, where $C$ is any path from (0,0) to (1,1), is independent of path.

15. Use Green's theorem to find the work done by the force field

$$F(x,y) = (x + 2y^2)\mathbf{j}$$

as the object moves once counterclockwise about the circle $(x-2)^2 + y^2 = 1$.

16. Evaluate surface integral $\iint_S \sqrt{1+4z}\, dS$ where S is the portion of the paraboloid $z = x^2 + y^2$ for which $z \leq 4$.

17. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F}.d\mathbf{R}$ where $\mathbf{F} = 2y\mathbf{i} - 6z\mathbf{j} + 3x\mathbf{k}$ and C is the intersection of the xy-plane and paraboloid $z = 4 - x^2 - y^2$, traversed counter clockwise as viewed from above.

18. Use the divergence theorem to evaluate $\iint_S \mathbf{F}.\mathbf{N}dS$ where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes with outward unit normal vector $\mathbf{N}$.

(This question paper contains 3 printed pages)

Sl No. of QP 8899

Roll No. 2019

Unique Paper Code : 235302 (22)

Name of the Paper : **MAHT-302—Numerical Methods and Programming**

Name of the Course : **B.Sc. (H) Mathematics**

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Semester : **III**

Duration : **3 Hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the **six** questions are compulsory.

Attempt **any two** parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of Scientific Calculator is allowed.

1. (a) Perform three iterations of Newton's method to find a root of the equation $x^3 - 5x + 1 = 0$, considering the starting approximation as 0.5.

(b) Let f be a continuous function on the interval [a,b] and suppose that f(a)f(b) < 0. Prove that the bisection method generates a sequence of approximations $\{p_n\}$ which converges to a root p ∈ (a, b) with the property

$$|p_n - p| \leq \frac{b-a}{2^n}$$

(c) Verify that the function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval (1, 2). Perform four iterations of the bisection method.

(13)

2. (a) Verify that the equation $x^5 + 2x - 1 = 0$ has a root in the interval (0, 1). Perform three iterations of the secant method to approximate a root, considering $p_0 = 0$ and $p_1 = 1$.

(b) Perform three iterations of the false position method to approximate a root of the function $f(x) = cos(x) - x$ in the interval (0,1).

(c) Define order of convergence of an iterative method. Find the order of convergence of Newton's method. (13)

3\. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

and use it to solve the system $AX = [4 \quad 4 \quad 6]^T$.

(b) Starting with the initial vector $X^{(0)} = (0,0,0)$, perform three iterations of the Gauss Seidal method to solve the system of equations, for the given coefficient matrix and the right hand side vector

$$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 4 & 2 \\ 2 & 1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}.$$

(c) Starting with the initial vector $X^{(0)} = (0,0,0)$, perform three iterations of the Jacobi method to solve the system of equations, for the given coefficient matrix and the right hand side vector

$$\begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \quad \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}.$$ (13)

4\. (a) Use Newton Divided difference Method to estimate $Sin(0.15)$ from the following data set

| | | |
|---|---|---|
| $x$ | 0.1 | 0.2 |
| $f(x) = Sin(x)$ | 0.09983 | 0.19867 |

and also verify the theoretical error bound.

(b) Find the Lagrange interpolation polynomial that fits the data:
$f(-1) = -2,\ f(1) = 0,\ f(4) = 63,\ f(7) = 342.$

Hence interpolate at $x = 5.0$.

(c) Prove that for $n+1$ distinct nodal points $x_0, x_1, x_2, \cdots, x_n$ there exists a unique interpolating polynomial of at most degree $n$. (12)

5. (a) Define the central difference operator (δ) and backward difference operator (∇).

Also prove that: $\nabla = -\frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$.

(b) If $f(x) = e^{ax}$, then show by induction method that $\Delta^n e^{ax} = (e^{ah} - 1)^n e^{ax}$.

(c) Derive the following backward difference approximation formula for the first order derivative, where $h$ is the spacing between the points.

$$f'(x_0) = \frac{1}{2h}(3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h))$$

(12)

6. (a) Evaluate $\int_1^2 \frac{dx}{x}$ by Trapezoidal Rule and verify the theoretical error bound.

(b) Apply Euler's method to approximate the solution of the initial value problem

$\frac{dx}{dt} = \frac{t}{x}, 0 \le t \le 3, \quad x(0) = 1, \quad h=0.5$

(c) Verify that the forward difference approximation:

$$f'(x_0) = \frac{1}{2h}(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h))$$

for the first order derivative provides the exact value of the derivative, regardless of the value of $h$, for the functions $f(x) = 1, \; f(x) = x, \; f(x) = x^2$ but not for the function $f(x) = x^3$.

(12)

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**SET B**

2019

Roll No.

This question paper contains 2 printed pages (23)

Sl. No. of Q.P.: 8900

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Unique paper code : 235304

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : MAHT 303 -- Algebra-II

Semester : III

Duration : 3 Hours

Maximum marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt any *two* parts from each question.
3. **All** questions are compulsory.

1. (a) Prove that $Z_n$={0,1,...,n-1} is a group under the addition modulo n.

   (b) Let H be a nonempty subset of a group G. Then prove that if $ab^{-1} \in H$ for all $a, b \in H$, then H is a subgroup of G.

   (c) Define Center $Z(G)$ of a group G. Also, prove that $Z(G)$ is a subgroup of G.

(6, 6, 2+4)

2. (a) In a group $G$, prove the following :

   (i) the identity element of G is unique,

   (ii) the cancellation laws hold in G.

   (b) Let $a$ be an element of a group $G$ such that $|a|$, the order of $a$, is finite. If $a^k$= e, then prove that $|a|$ divides k.

   (c) Define a cyclic group. Let $a$ and $b$ be elements in a group $G$ with $|a|$ = m, $|b| = n$ . If $(m, n) = 1$, then prove that $< a > \cap < b > = \{e\}$.

(3+3, 6, 2+4)

3. (a) Let $\alpha = (a_1, a_2 ... a_m)$ and $\beta = (b_1, b_2 ... b_k)$ be disjoint cycles. Then prove that $\alpha\beta = \beta\alpha$.

(b) Prove that the order of a permutation of a finite set written in disjoint cycles form is the LCM of the lengths of the cycles.

(c) State and prove Orbit Stablizer Theorem.

(6, 6, 2+4)

4. (a) Let $H$ be a subgroup of $G$ and let $a$ and $b$ be elements in $G$. Then, prove that

(i) $|aH| = |bH|$,

(ii) $aH = Ha$ if and only if $H = aHa^{-1}$.

(b) Prove that every subgroup of an Abelian group is normal. Is the converse true? Justify.

(c) Let N be a normal subgroup of a group G with order 2. Then prove that N is contained in the center Z(G) of G.

(6.5, 3.5+3, 6.5)

5. (a) Prove that the alternating group $A_n$ is a normal subgroup of $S_n$.

(b) Let $G$ be a finite Abelian group and let $p$ be a prime that divides the order of $G$. Prove that $G$ has an element of order $p$.

(c) Find the factor group $Z/4Z$.

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(6.5, 6.5, 6.5)

6. (a) Find all homomorphisms from $Z_{12}$ to $Z_{30}$.

(b) Let $\theta$ be a group homomorphism from $G$ to K. Let $H$ be a subgroup of $G$. Prove that

(i) $\theta(H)$ is a subgroup of K.

(ii) If $H$ is Abelian, then $\theta(H)$ is also Abelian.

(c) Let $\theta$ be a group homomorphism from $G$ to $K$ and let $g \in G$. If $\theta(g) = g'$, then prove that $\theta^{-1}(g') = gKer\,\theta$.

(6.5, 6.5, 6.5

This question paper contains 3 printed pages.

(24)

Roll No. 2019

S. No. of Question Paper: 8901

Unique Paper Code : **235501**

Name of the paper : **MAHT – 501: Differential Equations III**

Name of the course : **B.Sc. (Hons.) Mathematics**

Semester : **V**

Duration: **3 Hours** Maximum Marks: 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any three parts from each question.

1) (a) Solve the initial value problem using Laplace transform: (6)

$$y'' - 3y' + 2y = 4t + 3e^{t}, \quad y(0) = 1, y'(0) = -1.$$

(b) (i) Find the inverse Laplace transform of (2)

$$\frac{2s^2 - 4}{(s+1)(s-2)(s-3)}$$

(ii) Show that (2)

$$L\{Sin\,at\} = \frac{a}{s^2 + a^2}$$

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(iii)Find the inverse Laplace transform of (2)

$$F(s) = \frac{1}{s^4 - 16}$$

(c) Find the power series solution in powers of $(x-1)$ of the initial value problem

$$xy'' + y' + 2y = 0 \quad y(1) = 1, \quad y'(1) = 2$$ (6)

(d) Show that $x = 0$ and $x = -1$ are singular point of the differential equation $x^2(x+1)^2 y'' + (x^2 - 1)y' + 2y = 0$ where $x = 0$ is an irregular singular point and $x = -1$ is a regular singular point. (6)

2. (a) Starting from the number 1009 generate 10 random numbers using middle – square method. Also write drawbacks of middle square method. (6)

(b) Using Monte Carlo Simulation, write an algorithm (6)

$$y = Sin\, x \; over \; 0 \leq x \leq \frac{\pi}{2} \; with \; 0 \leq Sin\, x \leq 2$$

(c) Using graphical analysis

$$\text{Maximize } z = 2x + 3y$$

*subject to*

$$x + 2y \leq 6$$
$$2x + y \leq 16$$
$$x \geq 0,\; y \geq 0$$

(6)

(d) Using simplex method

$$\text{Minimize } z = 25x + 12y$$

*subject to*

$$5x + 22y \leq 14$$
$$5x + 3y \leq 17$$
$$x \geq 0,\; y \geq 0$$

(6)

3. (a)(i) What is the relative numbers of labelled and unlabelled graph with 3 vertices. Show with the help of graph. (3)

(ii) Draw a connected graph with 8 verities and a disconnected graph with 8 vertices and 2 components. (3)

(b) (i) Prove that in any balanced signed graph every cycle has an even number of edges. (4)

(ii) Define simple graph. Give an example of simple graph. (2)

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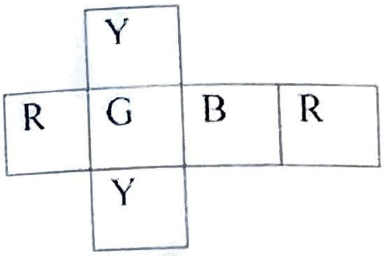
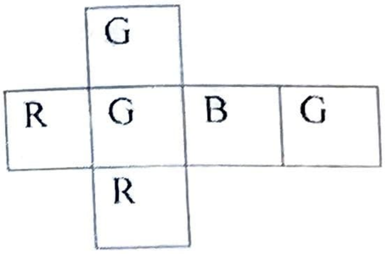
(c) Determine whether the given four cubes having four colours, can be stacked in a manner so that each side of the stack formed will have all the four colours exactly once. (6)



Cube 1



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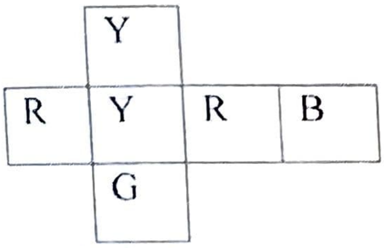

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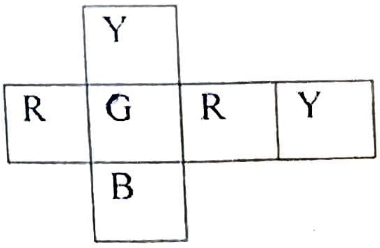

Cube 4

(d) Prove that a connected graph is semi-Eulerian iff it has exactly two vertices of odd degree. (6)

4) (a)Use the factorization:

$$s^4+4a^4=\left(s^2-2as+2a^2\right)\left(s^2+2as+2a^2\right)$$

And apply inverse Laplace transform to show that: (7)

$$L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}=\frac{1}{2a^2}\sin h\,at\sin at$$

(b) Using Frobenius series method, solve the differential equation

$x(1-x)y''-3xy'-y=0$ (7)

(c) A small harbour has unloading facilities for ships. Only one ship can be unloaded at any one time. The unloading time require for a ship depends on the type and the amount of cargo and varies from 45 to 90 minutes.

Below is given a situation with 5 ships:

| | Ship 1 | Ship 2 | Ship 3 | Ship 4 | Ship 5 |
|---|---|---|---|---|---|
| Time between successive ships | 20 | 30 | 15 | 120 | 25 |
| Unload time | 55 | 45 | 60 | 75 | 80 |

(i) Draw the time-line diagram depicting clearly the situation for each ship, the idle time for harbour and the waiting time. (5)

(ii) List the waiting times for all the ships and find the average waiting time. (2)

(d) Prove that there is no knight's tour on a 3 x 6 chessboard. (7)

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[This question paper has 7 printed pages]

2019

Sr. No. of Question Paper : 1298 (25)

Unique Paper Code : 203161

Name of the Paper : English Higher Qualifying

Name of the Course : B.A (hons)/B.Sc.(hons.) Mathematics

Semester : I/III

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Duration: 3 hours

Maximum Marks: 100

Instructions for candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.

SECTION I

1. Read the passage and answer the questions that follow:

Forests are among the top natural resources given to mankind. They provide us with both tangible and intangible resources without which the existence of many living things would be threatened.

To fulfil the rising demands for housing, cultivation among other human needs forests are being destroyed at an alarming rate. By cutting down trees, animals are forced to venture into regions of human habitat endangering not only their lives but the humans safety. A long lasting solution needs to be therefore formulated for a sustainable solution.

Forests have been important since ancient times. Forests covered 60 percent of the earth but with the rising population, extensive areas of forests have been cleared to allow farming, roads, mining, and other activities. In the present day only about 30 percent of forests, cover the earth.

Forests play an important role in improving the quality of the environment. They help to keep the environment healthy, clean, and beautiful. Trees release oxygen in the air and take in carbon dioxide, which if let to accumulate, could turn fatal to the human existence. Trees prevent soil erosion and reduce floods.

Strict measures need to be made to punish those who violate the set rules. Forests are vital for human existence and if nothing is done to conserve them all living things are in danger of extinction.

a. Answer whether the following are true or false: (5*1)

i. Forest is a man made resource.

ii. We cannot live without forests.

iii. Trees give us air to breathe.

iv. The forests make up 60% of the total world area.

v. Trees give us oxygen.

b. Match the words in Column 1 with their meanings in Column 2: (5*1)

| | Column 1 | Column 2 |
|---|---|---|
| i. | Ancient | large |
| ii. | Extensive | risking |
| iii. | Alarming | old |
| iv. | Endangering | worrying |
| v. | Reduce | decrease |

c. Answer the following questions in 2 or 3 sentences: (4*3)

i. What do forests provide us with?

ii. How do forests increase the quality of the environment?

iii. Why do wild animals move towards human habitat?

iv. Why are forests being cut?

2. Read the following passage and answer the questions that follow: (4*2)

Vijaya: Oh, Mr, Roy! I didn't hear you come in. That was Mrs Mukherjee.

Roy: So I heard. She's a good lady, but she always calls at the wrong time. Vijaya, we have a problem. Damodar says my car won't start........the other cars are out, and we have to meet Dr. Dass at the airport.

Vijaya: Shall I call a taxi?

Roy: No, Vijaya. Call Mr. Patil. After all, he did offer to help.

i. Where is Mrs. Mukherjee calling from?

ii. Why is the car not working?

iii. Who is Roy meeting at the airport?

iv. Why is Patil interested in helping Mr. Roy?

3. Answer any four of the following questions in about 40-60 words each: (4*3)

i. Why was David sent to Trivandrum by Mr. Roy?

ii. How do Shiva and David Blake know each other?

iii. What role does Vayu play in the text *The Tiger's Eye*?

iv. Mrs Mukherjee is the recipient of a special award? What is it?

v. What is the role of Gurusamy in the play?

SECTION II

4. Change the following into reported speech: (4*2)

i. He said, " I've lived here for a long time."

ii. Sheela said to her teacher, " May I go out now."

iii. David said to the guide, "Do you speak English?"

iv. He said to the Policeman, " I was playing football when the accident occurred."

5. Complete the dialogues: (5*2)

Rama: Where are you going this summer?

Shyam: ______________________ to Paris.

Rama: Oh that's wonderful. ______________________?

Shyam: I plan to be there for 10 days.

Rama: Who else is going with you.

Shyam: ______________________

Rama: Have you made a list of places you plan to visit?

Shyam: ______________________

Rama: That sounds fun. Hope you have a great time.

Shyam: ______________________

SECTION III

6. Fill in the blanks with the correct form of the verbs given: (8*1)

i. Maya ________ for you for over an hour.(wait)

ii. It is not worth ____________ so much money for this play. (pay).

iii. When I reached the airport, the entry gates __________ (close).

iv. I __________ Jaipur last month. (visit).

v. The criminal __________ the victim with a blunt object. (attack).

vi. His company is greatly ______________ after.(seek)

vii. His courage ______________ him (forsake).

viii. The terrified people ______________ to the mountains. (flee).

Fill in the blanks with the correct article (a, an or the): (9*1)

i. Gold is ............................ precious metal.

ii. Honest men speak ............................ truth.

iii. He looks as stupid as ............................ owl.

iv. I would like to meet Brad Pit, ...................... actor.

v. What did you do with ...................... camera I lent you?

vi. He is going out with ............................ German girl.

vii. He remained ............................ bachelor all his life.

viii. 'What is that noise?' 'I think it is ............................ helicopter.'

ix. The poor man fell asleep ______________ a tree.

8. Fill in the blanks with the contracted form of the underlined words: (8*1)

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i. You should not talk so much.

ii. They have written the text.

iii. Let us go home.

iv. He did not play cards.

v. I could not find my pen.

vi. Here is your book.

vii. I would ask him.

viii. Who is this girl?

9. Fill in the blanks with the correct preposition given: (8*1)

1. I slept ......................... nine o'clock.

to

till

until

2. I commenced work ......................... 1st May.

since

from

for

3. We walked ......................... the end of the street

till

to

for

4. The child has been missing ......................... yesterday.

since

from

for

5. He traveled seventy miles ......................... two hours.

for

in

by

6. I received this message ..................... 7 o'clock ................ the morning.

in, at

at, at

at, in

7. I saw him felling a big tree ........ ......... a hatchet.

by

with

off

8. An old feud existed .................. the two families.

between

among

within

10. Correct the following sentences(any five): (5*2)

i. It is raining when I got home last night.

ii. My sister is annoying today, but usually she is nice.

iii. I have not ate anything today.

iv. If I am a child, I would play outside.

v. Everyone have seen that movie.

vi. If we will be late, they will be angry.

vii. My father is thinking that I should stop smoking.

viii. Look! It is snow.

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[This question paper has 4 printed pages]

Sr. No. of Question Paper : 1299 (26) Roll no. 2019

Unique Paper Code : 203162

Name of the Paper : English lower Qualifying

Name of the Course : B.A (hons)/B.Sc.(hons.) Mathematics

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Semester : I/III

Duration: 3 hours Maximum Marks:100

Instructions for candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.

SECTION 1

1. Read the passage and answer the questions that follow:

The Narmada, also called Rewa is a river in central India and the fifth largest river in the Indian subcontinent. It is the third largest river that completely flows within India after Ganges and Godavari. It forms the traditional boundary between North India and South India and flows westwards over a length of 1,312 km before draining through the Gulf of Cambey (Khambat) into the Arabian Sea. It is one of only three major rivers in peninsular India that runs from east to west it is the largest west flowing river.

The River Narmada flows through a gorge of Marble rocks in Bhedaghat in Jabalpur, India. The water fall is called the Duandhara, the fall of mist, it flows for 3 km in a deep narrow channel through the magnesium limestone and basalt rocks called the Marble Rocks.

The source of the Narmada is a small tank called Narmada Kund located on the Amarkantak hill in Madhya Pradesh. The river descends from the Amarkantak hill range at the Kapildhara.

The Narmada happens to be one of the most sacred of the five holy rivers of India; According to a legend, the river Ganges is polluted by millions of people bathing in it. To cleanse herself, Ganges acquires the form of a black cow and comes to the Narmada to bathe in its holy waters. Legends also mention that the Narmada River is older than the river Ganges.

a. Answer the following in one or two sentences each: (5*2)

i. What is the other name for river Narmada?

ii. How large is this river?

iii. What is the source of the river?

iv. What is the meaning of 'Duandhara'?

v. How many holy rivers are there in India?

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b. State whether the following are true or false: (5*2)

1. The Narmada is the longest river in India.
2. The river descends from the Amarkantak hill range.
3. The River Narmada flows through a gorge of Marble rocks in Bhedaghat in Bhopal.
4. Narmada flows into the Arabian Sea.
5. Ganga remains the clean river.

c. Make sentences with any five words: (5*2)

Subcontinent, source, polluted, cleanse, traditional, sacred, descend.

d. Find antonyms of any five of the following words from the passage: (5*2)

Ascend, smallest, clean, minor, after, modern, goes

SECTION II

2. Change the following into indirect speech: (5*2)

   i. The teacher said, "I will take your class."

   ii. Rohit said to Amina, " I am going to Banglore tomorrow."

   iii. Sofia said to her mother, "I am hungry."

   iv. The old man said, "I was sleeping when my house was robbed."

   v. The little boy said to the policeman, "Can you take me home."

3. Fill in the blanks with the correct form of the verb given in the bracket. (5 * 2)

   i. Last Sunday I ______ for a movie. (go)

   ii. The Indian team ______ their victory yesterday (celebrate)

   iii. Rajesh ______ to go to England for higher studies. (plan)

   iv. We must hurry otherwise we will ______ the plane. (miss)

   v. We ______ Delhi early in the morning tomorrow.(reach)

4. Fill in the blanks with suitable articles (a, an, the): (5*2)

   i. I don't know where to go for ______ vacation.

   ii. ______ apple______ day keeps the doctor away.

   iii. ______ cat has drunk all the milk.

   iv. We have ______ best team.

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5. Complete the following sentences with infinitives (to+verb): (5*2)

   i. I like ______

   ii. They want ______

   iii. We promise ______

iv. My mother advised me ____________

v. The president promised ____________

## SECTION III

6. Write a letter to your brother who is in hostel telling him the importance of taking good diet.

Or

Write an application to the Principal of your college requesting her to increase the library timings. (20)

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